

1.1

Patterns and Inductive Reasoning 5

- Goals**
- Find and describe patterns.
 - Use inductive reasoning to make conjectures.

VOCABULARY

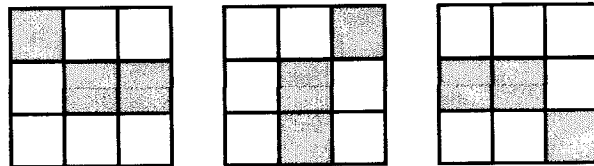
Conjecture

Inductive reasoning

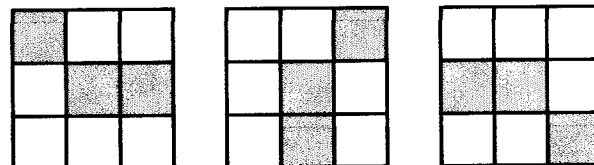
Counterexample

Example 1 Describing a Visual Pattern

Sketch the next figure in the pattern.



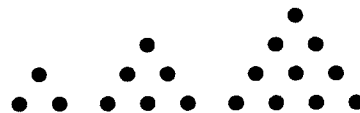
Solution



How does the shading change from one figure to the next? Is the figure rotated?

Checkpoint Sketch the next figure in the pattern.

1.



Example 2**Describing a Number Pattern**

Describe a pattern in the sequence of numbers. Predict the next number.

a. 128, 64, 32, 16, ...

b. 5, 4, 2, -1, ...

Solution

a. Each number is _____ the previous number. The next number is ____.

b. Subtract ____ to get the second number, then subtract ____ to get the third number, then subtract ____ to get the fourth number. To find the fifth number, subtract ____ from the fourth number.

Answer So, the next number is ____ - ____, or ____.

✓ **Checkpoint** Describe a pattern in the sequence of numbers. Predict the next number.

2. 4, -20, 100, -500, ...

3. 10, 20, 40, 70, 110, ...

Example 3**Making a Conjecture**

Complete the conjecture.

Conjecture: The sum of the first n even positive integers is ____? ____.

Solution List some specific examples and look for a pattern.

Examples:

first even integer: $2 = 1(\underline{\quad})$

sum of first **two** even positive integers: $2 + 4 = \underline{\quad} = 2(\underline{\quad})$

sum of first **three** even positive integers: $2 + 4 + 6 = \underline{\quad} = 3(\underline{\quad})$

sum of first **four** even positive integers: $2 + 4 + 6 + 8 = \underline{\quad} = 4(\underline{\quad})$

Conjecture: The sum of the first n even positive integers is ____.

Show the conjecture is false by finding a counterexample.

Conjecture: If the difference of two numbers is odd, then the greater of the two numbers must also be odd.

Solution

Counterexample: $\underline{\quad} - \underline{\quad} = \underline{\quad}$

So, the conjecture is $\underline{\quad}$.

Write a difference so that the greater of the two numbers is even.

✓ **Checkpoint** Complete the following exercises.

4. Complete the conjecture based on the pattern you observe.

$$1 = 1$$

$$1 + 2 = 3 = \frac{2(2 + 1)}{2}$$

$$1 + 2 + 3 = 6 = \frac{3(3 + 1)}{2}$$

$$1 + 2 + 3 + 4 = 10 = \frac{4(4 + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 = 15 = \frac{5(5 + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 + 6 = 21 = \frac{6(6 + 1)}{2}$$

Conjecture: The sum of the first n positive integers

is $\underline{\hspace{2cm}}$.

5. Show the conjecture is false by finding a counterexample.

Conjecture: The difference of two negative numbers is always negative.

Practice A

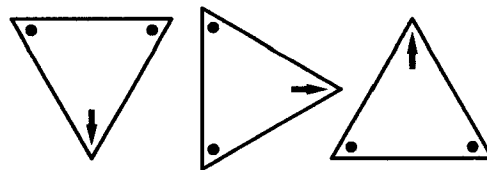
For use with pages 3–9

Sketch the next figure in the pattern.

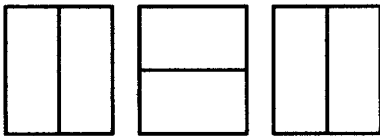
1.



2.



3.



4.



Describe a pattern in the sequence of numbers. Predict the next number.

5. 2, 5, 8, 11, ...

6. 27, 9, 3, 1, ...

7. 123, 234, 345, 456, ...

8. 5, 7, 11, 17, 25, ...

9. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

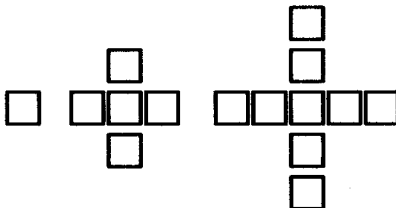
10. $\frac{5}{4}, \frac{4}{6}, \frac{3}{8}, \frac{2}{10}, \dots$

11. 4, 1, -2, -5, ...

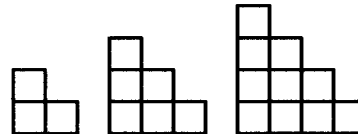
12. 1, 4, 9, 16, ...

The first three objects in a pattern are shown. How many squares are in the next object?

13.



14.



Complete the conjecture based on the pattern you observe in the specific cases.

15. The product of an odd number and an even number is ____?

16. The sum of an odd number and an even number is ____?

$3 \cdot 8 = 24$

$6 \cdot 5 = 30$

$17 + 22 = 39$

$8 + 37 = 45$

$11 \cdot 24 = 264$

$102 \cdot 31 = 3162$

$135 + 48 = 183$

$94 + 85 = 179$

Algebra Review

Adding and Subtracting Integers

Adding Integers

Same signed integers-

Keep the sign of _____ the numbers and _____.

Different signed integers (one positive, one negative)-

Keep the sign of the _____ number, then _____.

Subtracting Integers

Change the minus sign to a plus sign and switch the sign of the number after the new plus sign.

Subtract to find the distance on a number line if given two points on a number line.

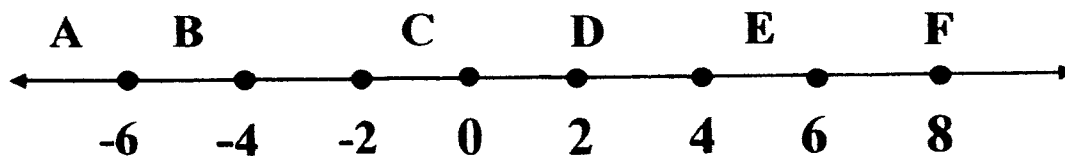
If given two points on a number line-



1. Subtract the two numbers.
2. Take the absolute value.

Absolute Value: The distance a number is away from zero on the number line.

Absolute values are ALWAYS POSITIVE



The coordinate of B

The distance of \overline{AD}

The graph of A

The distance of \overline{BD}

The coordinate of F

The distance of \overline{CF}

The length between A and F

The distance of \overline{EB}

1.2 Points Lines and Planes

Object	Example	Name	Symbol
A <u>point</u> is a location in space.			
A <u>line</u> is made up of infinitely many points it extends in both directions forever.			
A <u>line segment</u> is part of a line. It has two endpoints.			
A <u>ray</u> is part of a line. It has one endpoint.			
A <u>plane</u> is a flat surface. It extends endlessly in all directions.			

Write the definitions of the following terms:

Collinear

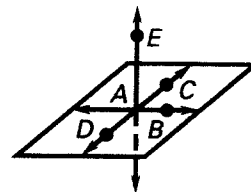
Coplanar

Opposite Rays

Intersection

Example 1 Naming Collinear and Coplanar Points

- Name three points that are collinear.
- Name three points that are coplanar.
- Name four points that are not coplanar.

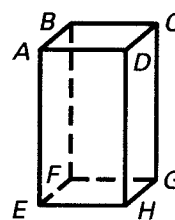


Solution

- Points , , and lie on the same line, so they are collinear.
- There are many correct answers. For instance, points , , and lie on the same plane. Also, points , , and are coplanar, although the plane containing them is not drawn.
- There are many correct answers. For instance, points , , , and do not lie on the same plane.

Checkpoint Complete the following exercises.

- Name three points in the diagram that are not collinear.

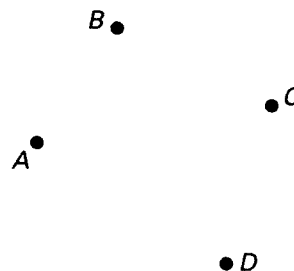


- Name the point in the diagram that is coplanar with points A, D, and E.

Example 2 Drawing Lines, Segments, and Rays

Draw four noncollinear points, A, B, C, and D. Then draw \overline{AB} , \overrightarrow{BC} , \overleftrightarrow{CD} , \overrightarrow{DA} , and \overrightarrow{BD} .

A, B, C, and D are shown.



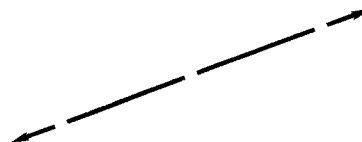
- Draw \overline{AB} .
- Draw \overrightarrow{BC} .
- Draw \overleftrightarrow{CD} .
- Draw \overrightarrow{DA} .
- Draw \overrightarrow{BD} .

Example 3 *Drawing Opposite Rays*

Draw a line. Label three points on the line and name a pair of opposite rays.

Draw points X, Y, and Z on the given line so that Y is between X and Z.

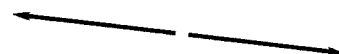
The opposite rays are _____ and _____.

**Example 4** *Sketching Intersections*

Sketch two lines that do not intersect and a line that intersects each of the other lines.

Draw a line that does not intersect the given line.

Then draw a third line that intersects the first two lines. Emphasize the points of intersection.



Practice A

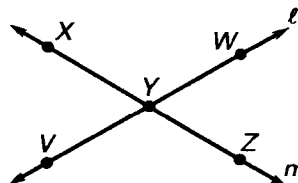
For use with pages 10–16

SHOW WORK & ANSWERS ON
NOTEBOOK PAPER**Draw a sketch and label as needed.**

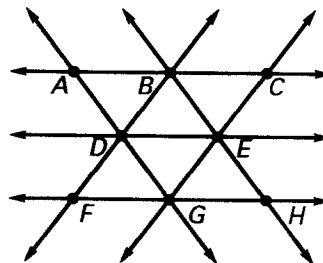
- Three collinear points, A , B , and C .
- \overleftrightarrow{MN} intersecting \overleftrightarrow{PQ} at point R .
- Coplanar points W , X , Y , and Z .
- Opposite rays, \overrightarrow{JK} and \overrightarrow{JC} .

Decide whether the statement is true or false.

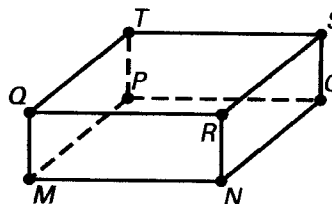
- Point X lies on line m .
- X , Y , and Z are collinear.
- Point W lies on line m .
- X , Y , and Z are coplanar.
- Point V lies on line l .
- V , Y , and X are collinear.
- Point Y lies on line l .
- V , Y , and X are coplanar.

**Name a point that is collinear with the given points.**

- B and E
- F and H
- D and G
- A and C
- H and E
- G and C
- A and D
- B and C

**Name a point that is coplanar with the given points.**

- M , N , and O
- M , N , and R
- T , Q , and M
- T , Q , and R
- T , S , and R
- T , S , and O
- O , S , and R
- O , P , and M

**In Exercises 29–34, complete the sentence.**

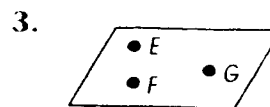
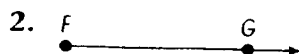
- Collinear points are points that ____.
- Coplanar points are points that ____.
- \overline{XY} consists of the endpoints X and Y and all points on the line \overleftrightarrow{XY} that lie ____.
- \overrightarrow{MN} consists of the initial point M and all points on the line \overleftrightarrow{MN} that lie ____.
- Two rays or segments are collinear if they ____.
- \overrightarrow{PQ} and \overrightarrow{PT} are opposite rays if ____.
- Explain the difference between \overrightarrow{BC} and \overrightarrow{CB} .

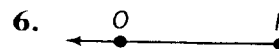
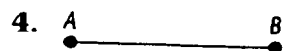
Practice • Points, Lines, and Planes

2

Lesson 1.1

Name each figure.





Draw each figure.

7. line UV

8. point M

9. line segment OP

10. ray JK

11. \overleftrightarrow{FG}

12. \overline{XY}

#11

POINT, LINE, AND PLANE POSTULATES

- Postulate 5** Through any two points there exists exactly one line.
- Postulate 6** A line contains at least two points.
- Postulate 7** If two lines intersect, then their intersection is exactly one point.
- Postulate 8** Through any three noncollinear points there exists exactly one plane.
- Postulate 9** A plane contains at least three noncollinear points.
- Postulate 10** If two points lie in a plane, then the line containing them lies in the plane.
- Postulate 11** If two planes intersect, then their intersection is a line.

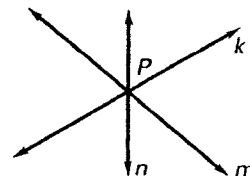
Example 3 Using Postulates and Counterexamples

Decide whether the statement is *true* or *false*. If it is false, give a counterexample.

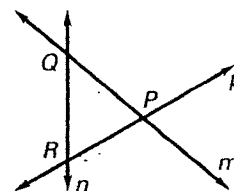
- A point can lie on more than two lines.
- Three lines can intersect at no more than three distinct points.
- If two lines are coplanar, then they intersect.

Solution

- a. In the diagram at the right, point P is the intersection of line k , line m , and line n . So, it is true that a point can lie on more than two lines.



- b. In the diagram at the right, line k and line m intersect at point P , line m and line n intersect at point Q , and line k and line n intersect at point R . There are no more possible intersections. So, it is true that three lines can intersect at no more than three distinct points.



- c. In the diagram at the right, line m and line n are coplanar, but they do not intersect. So, it is false that if two lines are coplanar, then they intersect.



#11

HW : P 13-14 8-42 even
P 24 60-70 all

Unit 1.2 Midpoints

Definitions

Congruent Line Segments: (symbol for congruence \cong)

Midpoint of Line Segments:

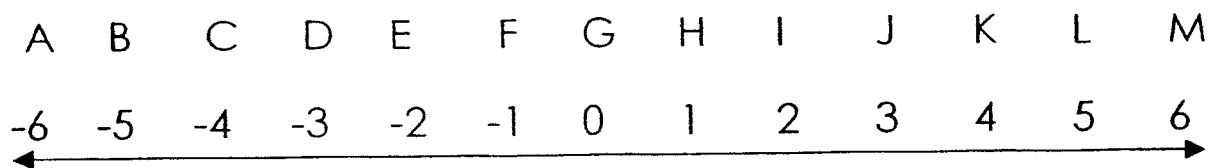
Bisector:

Examples of Congruence

\cong

Examples of Midpoints

Lengths of line segments



$GL =$

$CI =$

$AE =$

$BL =$

$EJ =$

$BK =$

$DJ =$

$AD =$

$\text{Midpoint of GI} =$

$\text{Midpoint of AM} =$

$\text{Midpoint of DI} =$

$\text{Midpoint of DL} =$

$\text{Coordinate of E} =$

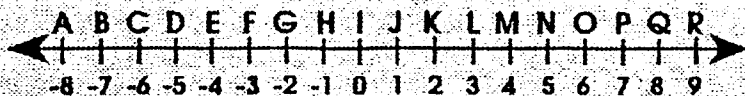
$\text{The graph of } 4 =$



Segments and Angles

Lengths and Midpoints of Segments

Name the point or state the number described.



The coordinate of K is 2.

The length of \overline{CG} is 4.

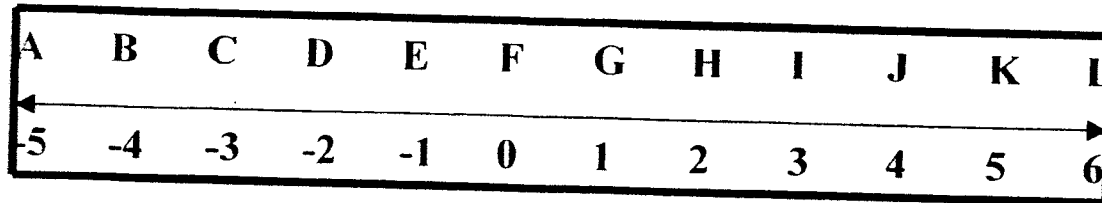
The graph of the number 7 is P.

The midpoint of \overline{RH} is M.

1. The length of \overline{DJ}
2. The coordinate of B
3. The endpoint of \overrightarrow{FJ}
4. The midpoint of \overline{HP}
5. The point on \overrightarrow{CF} two units from C
6. The distance between D and Q
7. Two points three units from E
8. The midpoint of \overline{BP}
9. The coordinate of the origin
10. The graph of the number 5
11. The length of \overline{GO}
12. The coordinate of M

Practice: Lengths and Midpoints of Segments

Name the point or state the number described.



1. The length of CK
2. The coordinate of J
3. The point on ray DJ two units from D
4. The midpoint of BH
5. The midpoint of GK
6. The midpoint of CK
7. The endpoint of ray BE
8. Distance between D and L
9. The graph of number -3
10. The midpoint of CG

1.3

Segments and Their Measures

14

- Goals**
- Use segment postulates.
 - Use the Distance Formula to measure distances.

VOCABULARY

Postulates

Coordinate

Distance

Length

Between

Distance Formula

Congruent segments

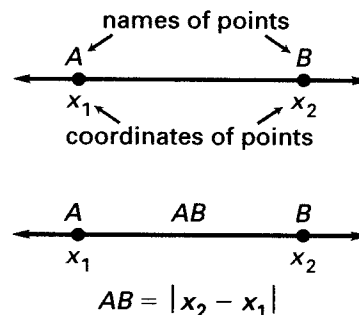
MIDPOINT

POSTULATE 1: RULER POSTULATE

The points on a line can be matched one to one with real numbers. The real number that corresponds to a point is the _____ of the point.

The _____ between points A and B , written as AB , is the absolute value of the difference between the coordinates of A and B .

AB is also called the _____ of \overline{AB} .

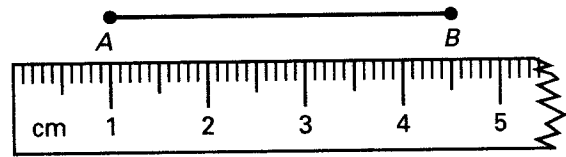


Example 1 Finding the Distance Between Two Points

Measure the length of the segment to the nearest tenth of a centimeter.

$$AB = | \quad - \quad |$$

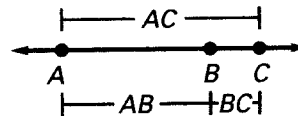
$$= \quad$$



Answer The length of \overline{AB} is about \quad centimeters.

POSTULATE 2: SEGMENT ADDITION POSTULATE

If B is between A and C , then $AB + BC = AC$. If $AB + BC = AC$, then B is between A and C .

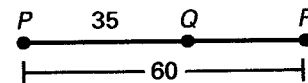


Use the Segment Addition Postulate to find the length.

3. Find JL .



4. Find QR .

**LESSON****1.3**

NAME _____ DATE _____

Practice A

For use with pages 17–25

Draw a sketch of the three collinear points. Then write the Segment Addition Postulate for the points.

7. S is between D and P .

8. J is between S and H .

9. C is between Q and R .

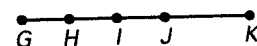
10. T is between M and N .

In the diagram of collinear points, $GK = 24$, $HJ = 10$, and $GH = HI = IJ$. Find each length.

11. HI

12. IJ

13. GH



14. JK

15. IG

16. IK

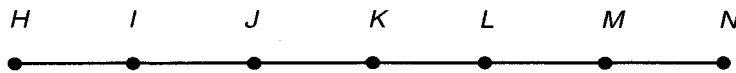
Study Guide

BB 1.3/1.5

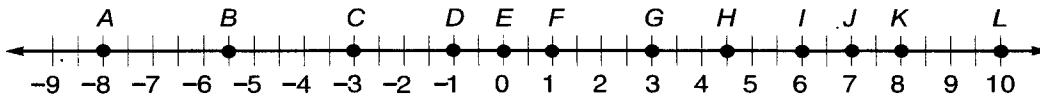
Congruent Segments

Two segments are congruent if they have the same length. The **midpoint** of a segment separates the segment into two congruent segments. To **bisect** a segment means to separate it into two congruent parts. The midpoint always bisects a segment.

Use the line to name all segments congruent to each given segment.

1. \overline{HM} 2. \overline{JL} 3. \overline{NJ} 4. \overline{HI}

Use the number line to name the midpoint of each segment.

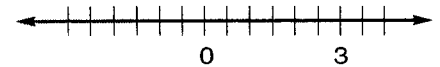
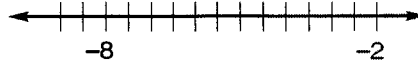
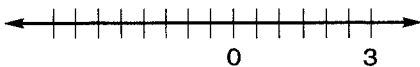
5. \overline{EI} 6. \overline{IL} 7. \overline{AK} 8. \overline{CF} 9. \overline{AC} 10. \overline{DL} 11. \overline{CG} 12. \overline{IK}

For each exercise below, the coordinates of points P and Q , respectively, are given. Graph P and Q . Then draw and label the coordinate of the midpoint of PQ .

13. -4 and 2

14. -9 and -5

15. -3 and 4



Skills Practice

Blue Book 1.3

Segments and Properties of Real Numbers

Three segment measures are given. The three points named are collinear. Determine which point is between the other two.

1. $XY = 15$, $AY = 31$, $AX = 46$

2. $AB = 12$, $BC = 20$, $AC = 32$

3. $MO = 75$, $MC = 34$, $OC = 41$

4. $DE = 58$, $GE = 12$, $DG = 70$

5. $HM = 2$, $JM = 1$, $HJ = 3$

6. $WX = 8$, $WA = 4$, $AX = 4$

Use the line to find each measure.



7. If $AC = 10$ and $CG = 21$, find AG .

8. If $AI = 72$ and $GI = 11$, find AG .

9. If $CG = 24$ and $EG = 14$, find CE .

10. If $AK = 80$ and $IK = 24$, find AI .

11. If $AC = 18$ and $CK = 72$, find AK .

12. If $CI = 65$ and $GI = 13$, find CG .

Find the length of each segment in centimeters and in inches.

13. _____

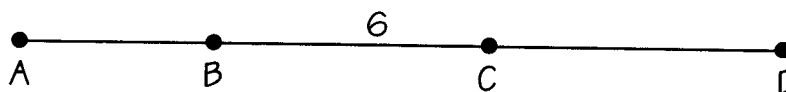
14. _____

15. _____

16. _____

How do you clean all the mice out of your pores?

Solve for the missing lengths. To figure out the joke, place the letter of each problem above the answer on the line(s) below. Some blanks will go unfilled.

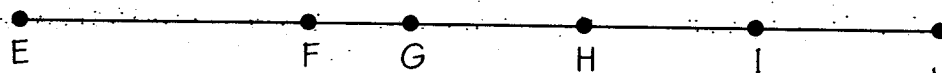


A: If $AB = 5$, $AC =$ _____

H: If $BD = 11$, $CD =$ _____

E: If $\overline{AB} \cong \overline{CD}$ and $AB = 3$, $AD =$ _____

T: If $\overline{AB} \cong \overline{CD}$ and $AD = 18$, $CD =$ _____



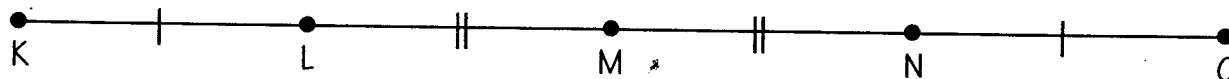
$EJ = 44$, $FG = 7$, $HI = 9$, $FI = 26$, $HJ = 13$

D: $GH =$ _____

I: $IJ =$ _____

S: $FJ =$ _____

U: $EF =$ _____



$NO = KL$ and $NO = 3$, $LO = 21$ and $LM = MN$

O: $KL =$ _____

P: $MN =$ _____

W: $KN =$ _____

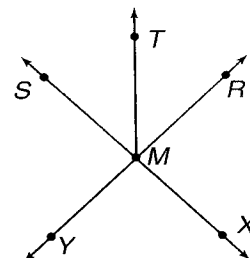
M: $KO =$ _____

21 4 6 5 8 11 15 24 3 14 30 12 18 9 11 10

Skills Practice**Points, Lines, and Planes**

Use the figure at the right to name examples of each term.

- four points
- two lines
- four segments
- one ray whose endpoint is M
- three collinear points
- one point that is *not* on \overleftrightarrow{YR}
- a segment with points T and M as its endpoints
- a line that does not contain R
- a line containing M
- a segment that lies on \overleftrightarrow{YR}



Determine whether each model suggests a point, a line, a ray, a segment, or a plane.

- | | |
|----------------------------------|---|
| 11. a toothpick | 12. a floor |
| 13. the tip of a pin | 14. the surface of the water in a swimming pool |
| 15. a beam of light from a laser | 16. fence pole |

Draw and label a figure for each situation described.

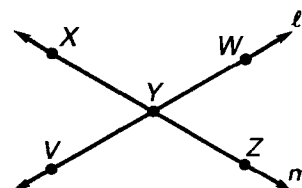
- | | |
|---|---|
| 17. point K lies on \overleftrightarrow{RT} | 18. plane \mathcal{H} contains line a |
| 19. \overleftrightarrow{AB} lies in plane \mathcal{M} containing point R not on \overleftrightarrow{AB} | 20. \overrightarrow{AX} and \overrightarrow{AY} such that point A is the only point common to both rays |

Practice B

For use with pages 10–16

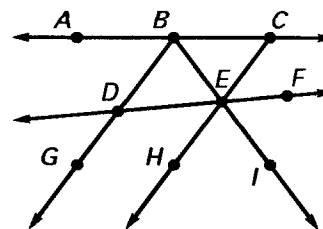
Decide whether the statement is **true** or **false**.

1. Point X lies on line m .
2. X , Y , and Z are collinear.
3. Point W lies on line m .
4. X , Y , and Z are coplanar.
5. \overrightarrow{YW} and \overrightarrow{YV} are collinear.
6. \overrightarrow{YW} and \overrightarrow{YV} are coplanar.
7. \overrightarrow{YX} and \overrightarrow{YV} are collinear.
8. \overrightarrow{YX} and \overrightarrow{YV} are coplanar.



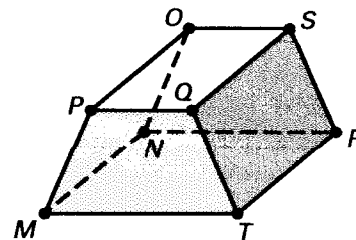
Name a point that is collinear with the given points.

9. B and E
10. C and H
11. D and G
12. A and C
13. H and E
14. G and B
15. B and I
16. B and C



Name a point that is coplanar with the given points.

17. M , N , and R
18. M , N , and O
19. M , T , and Q
20. Q , T , and R
21. T , R , and S
22. Q , S , and O
23. O , P , and M
24. O , S , and R



Complete the sentence.

25. \overline{AB} consists of the endpoints A and B and all points on the line \overleftrightarrow{AB} that lie ____.
26. \overrightarrow{PQ} consists of the initial point P and all points on the line \overleftrightarrow{PQ} that lie ____.
27. Two rays or segments are collinear if they ____.
28. \overrightarrow{MN} and \overrightarrow{ML} are opposite rays if ____.

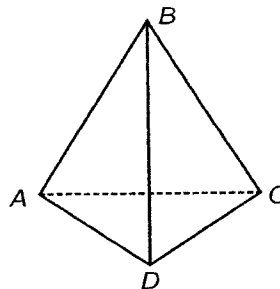
Sketch the figure described.

29. Three points that are coplanar but not collinear.
30. Three lines that intersect at a single point.
31. Three lines that intersect at two points.
32. Three lines that intersect at three points.
33. Two planes that intersect.
34. Two planes that do not intersect.
35. Two rays that intersect at their initial points.
36. Two rays that do not intersect.

Practice

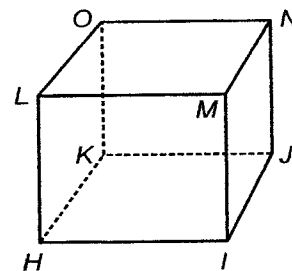
Postulates

1. Points A , B , and C are noncollinear. Name all of the different lines that can be drawn through these points.
2. What is the intersection of \overline{LM} and \overline{LN} ?
3. Name all of the planes that are represented in the figure.



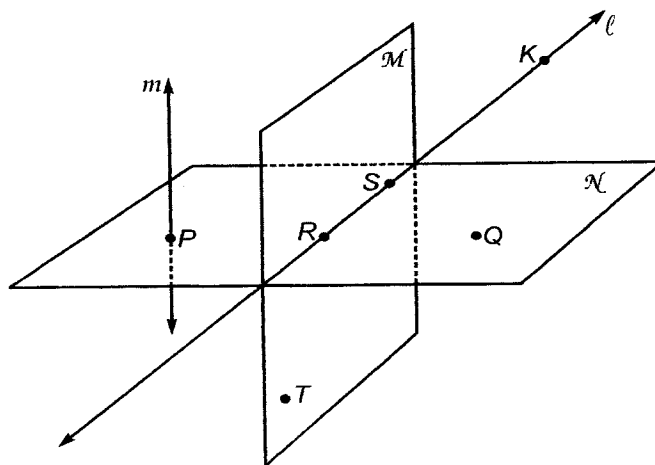
Refer to the figure at the right.

4. Name the intersection of ONJ and KJI .
5. Name the intersection of KOL and MLH .
6. Name two planes that intersect in \overline{MI} .



In the figure, P , Q , R , and S are in plane \mathcal{N} . Determine whether each statement is true or false.

7. R , S , and T are collinear.
8. There is only one plane that contains all the points R , S , and Q .
9. $\angle PQT$ lies in plane \mathcal{N} .
10. $\angle SPR$ lies in plane \mathcal{N} .
11. If X and Y are two points on line m , then \overline{XY} intersects plane \mathcal{N} at P .
12. Point K is on plane \mathcal{N} .
13. \mathcal{N} contains \overline{RS} .
14. T lies in plane \mathcal{N} .
15. R , P , S , and T are coplanar.
16. ℓ and m intersect.



Name: _____ Period _____

Review: Quiz Section 1.1-1.2 -1. 3

Points, Lines, and Planes

Directions: Write the letter of the correct definition on the line in front of the matching term.

- | | |
|------------------|--|
| 1. _____ Ray | A. A location in space |
| 2. _____ Segment | B. A flat surface; extends endlessly in all directions |
| 3. _____ Point | C. Part of a line; has 2 endpoints |
| 4. _____ Plane | D. Made up of infinitely many points; it extends in both directions forever. |
| 5. _____ Line | E. Part of a line; has one endpoint |

Midpoints of Line Segments

Point O is the midpoint of \overline{CR} . Find the indicated lengths.

$CA = 10$

6. $AR =$

7. $CR =$



T is the midpoint of \overline{MH} . A is the midpoint of \overline{MT} .

$\overline{MA} = 5$

8. $\overline{AT} =$

9. $\overline{TH} =$



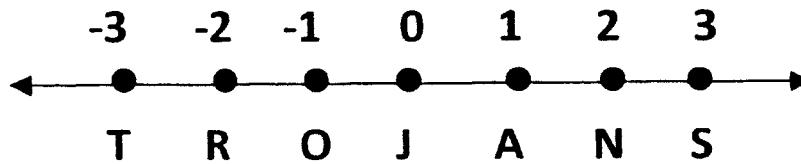


In the figure above, suppose $BL = 9$, $LU = 4$, $BE = 23$

10. Then $UE =$

11. Is U the midpoint of \overline{BE} ?

Lengths and Midpoints of Segments



Directions: Answer the following using the figure directly above.

12. The length of TS

16. The midpoint of RN (letter)

13. The coordinate of S

17. The endpoint of ray SA (number)

14. The point on ray OA
two units from O (letter)

18. Distance between R and O

15. The midpoint of TJ (number)

19. The graph of number 2